THE SIMULATION OF THE RELIABITY OF ASSETS USING THE MONTE CARLO METHOD. THE SPECIFIC-CASE OF COMPLEX AND COHERENT SYSTEMS K-OUT-OF-N, WITH CENSORED DATA

A SIMULAÇÃO DA FIABILIDADE DE ATIVOS UTILIZANDO O MÉTODO MONTE CARLO. O CASO ESPECÍFICO DE SISTEMAS COMPLEXOS E COERENTES K-OUT-OF-N, COM DADOS CENSURADOS

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AUTOR/A 1: Daniel Gaspar, IPV-ESTGV, Portugal, danigaspar@estgv.ipv.pt
AUTOR/A 2: Luís Ferreira, FEUP/UP, Portugal, iferreira@fe.up.pt

ABSTRACT

Management of the life cycle of physical assets is based on the estimated life of the equipment, whether during the project, installation and start-up or throughout its useful life. In many situations, engineering needs to calculate or simulate equipment's estimated useful life using accurate data, which is often censored or incomplete. In many cases, equipment, regarding failures, can be structurally represented as block diagram systems. For this article, complex and coherent systems (from a reliability point of view) were studied. Censored data usually results in the loss of important information but must be included in the reliability analysis models of these complex systems. The article develops the complex and coherent systems theory and the respective reliability models. With a new and original approach, simulation algorithms were developed to generate random and censored data (right-censored and type I data). Two case studies of complex systems were designed to validate the algorithms. For each case, a set of simulations was developed with the variation of the reliability models' different parameters to compare better, tune and optimize the simulation of these complex systems.

One of the relevant results shows that the more censored data in the sample, the greater the bias and error about the true value. The variation of the parameter β (shape factor) from β = 0:5 to β = 1:5 proportionally increases the bias.

This article aims to validate the use of the Monte Carlo simulation tool and the Weibull statistical distribution and contribute to improving, with more precision and speed, algorithms for simulating the reliability of complex and coherent systems in the presence of censored data.

KEYWORDS: Censored Data; Complex System and Coherent *k-out-of-n*; Monte Carlo Simulation; Reliability.

RESUMO

A gestão do ciclo de vidas dos ativos físicos, é feita com base estimativa de vida dos equipamentos, seja no projecto, na instalação e arranque ou ao longo da sua vida útil. Em muitas situações, a engenharia depara-se com necessidade de calcular ou simular o tempo estimado de vida útil dos equipamentos, usando dados reais, que muitas vezes são dados censurados ou incompletos. Em muitos casos os equipamentos, relativamente às falhas, podem ser estruturalmente representados como sistemas de diagramas de blocos. Para este artigo foram estudados os sistemas complexos e coerentes (do ponto de vista da fiabilidade). Os dados censurados, normalmente resultam em perda de informação importante, mas devem ser incluídos nos modelos de análise de fiabilidade destes sistemas complexos. O artigo desenvolve a teoria de sistemas complexos e coerentes e os respectivos modelos de fiabilidade. Com uma abordagem nova e original foram desenvolvidos os algoritmos de simulação com geração de dados aleatórios e censurados (dados censurados à direita e tipo I) Para validação dos algoritmos, foram desenvolvidos dois casos de estudos de sistemas complexos. Para cada um dos casos foi desenvolvido um conjunto de simulações com a variação dos diferentes parâmetros dos modelos de fiabilidade de forma a se comparar, afinar e optimizar melhor a simulação destes sistemas complexos.

Um dos resultados relevantes mostra que quanto mais dados censurados na amostra existem, maior é o enviesamento e o erro relativamente ao valor verdadeiro. A variação do parâmetro β (factor de forma) de β = 0:5 para β = 1:5 é o que proporcionalmente aumenta o enviesamento.

Este artigo pretende validar o uso da ferramenta de simulação monte Carlo e da distribuição estatística de weibull e dar um contributo para melhorar, com mais precisão e rapidez, os algoritmos para a simulação da fiabilidade de sistemas complexos e coerentes na presença de dados censurados.

PALAVRAS-CHAVE: Dados Censurados; Fiabilidade; Simulação Monte Carlo; Sistemas Complexos e Coerentes *k-ou-of-n*.

1. INTRODUCTION

Asset management, some authors state (Maletič et al., 2020), (Trindade et al., 2019) and (J. E. Amadi-Echendu, 2004), has the main goal of realizing the value of an organization's assets and has a significant impact on the performance of organizations. Its practice and implementation have several stages and activities, including the study, evaluation and management of the life cycle of equipment based on reliability.

The current era of 4.0 industry and digital transformation is characterized by the use of advanced technologies that allow means for monitoring and predicting the performance of assets and processes (Crespo Marquez et al., 2020), which enables the review of the maintenance programs and contributes to the reduced asset management costs, ease of maintenance, more safety and business risk mitigation.

The subject of reliability and simulation of systems and the structural relationship between a system and its components is very important in the field of reliability. A comprehensive discussion of reliability theory can be found in (Barlow & Proschan, 1975) and in (Kaufmann et al., 1977). Reliability Block Diagrams — RBD can be defined as a network of blocks describing the system's function with logical connections of components needed to produce a specified system function. Barlow (1981) presents an exhaustive description of the theory of RBD and, more recently, Rausand (2004). If the system has more than one function, each function must be considered individually, and different diagrams need to be made for each system function. The system is fixed in one moment of time; the present state of the system is assumed to depend only on the current states of the components. The connection through a block, in RBD, means that the component is functioning.

A Reliability Block Diagram is developed in terms of functions. Usually not have an account for safety and auxiliary functions and components used to protect equipment, people or the environment. Reliability Block Diagrams can be used for repairable and non-repairable systems or components. In the survival analysis and reliability field, there are several situations in which equipment, components, and units are lost or taken from the study while they are still working. The data censored may occur in control situations, as in life-testing and preassigned time or actual operations, and to make a predictive analysis of failures on time, with systems with vast numbers of sensors and monitoring lots of parameters; in this case, using reliability models containing censored data is fundamental. Genschel & Meeker (2010) refers to the fact that, in practice, life test data are almost always time-censored or type

I because the study defines the time at which the test will end. Balakrishnan et al. (2000) have more details about when the progressive censoring schemes take place. Several methods and techniques have been proposed over the past decades for analyzing different types of reliable data. Most of them refer to complete data. However, evaluating highly censored reliability data has not been widely studied. Nelson (1985) presented an excellent work on this topic. In the beginning, few of the studies used simulation tools, but over time, simulation in the reliability field increased, most of them to estimation parameters.

Olteanu & Freeman (2010) conducted a simulation study that compared the performance of maximum likelihood (ML) and median-rank regression (MRR) methods in estimating Weibull parameters for highly censored reliability data. In addition to the well-known large-sample optimal properties associated with ML estimators, experience, including many simulation studies, has shown that ML estimators are generally hard to beat consistently, even in small samples (Genschel & Meeker, 2010), (Somboonsavatdee et al., 2007). Recently, the estimation of parameters from different lifetime distributions based on progressive type-II censored samples has been studied by several authors, including (Balakrishnan & Kannan, 2001), (Mousa & Jaheen, 2002), (Childs & Balakrishnan, 2000) and (Soliman, 2005).

This article is concerned with analyzing the simulation of censored reliability data. Indeed, the loss of information resulting from the unavailable exact failure times will negatively impact the efficiency of reliability analysis. Many articles use the percentage of data censored (% C) to compare and analyze the model and study simulations, like in (Balakrishnan & Mitra, 2012), (Birolini, 2017) and (Ross, 2012). The use and application of data censored in the field of reliability can be seen in (Wang & Coit, 2005), (Horst, 2009). The type of distribution used in this study is typically used in the reliability field. Understanding and developing a systematic method to build an accurate simulation model in the presence of censored data is essential, giving more accuracy and precision to the simulation process in the reliability field (Gaspar, 2019) and (Nelson, 1985).

2. STATE OF ART: SYSTEMS, RELIABILITY AND CENSORED DATA

A. SYSTEM OF COMPONENTS

A system composed of n components will be classified as a system of order n. The component are to be numbered consecutively from *1 to n*. Let $C = \{c_1, c_2, \dots, c_n\})$ be the set of all components, where c_i is the i^{th} component, and n is the number of components in the system. Let x_i be the state of component c_i the system can be in one and only one of two states, that are either functioning or failed. To indicate the state of the i^{th} component a binary indicator variable x_i to component i is assigned:

$$x_i = \begin{cases} 1, & \text{if component i is functioning,} \\ 0, & \text{if component i is failed.} \end{cases}$$

for i=1,...,n, where n is the number of the components in the system.

The number of components n in the system is called the order of the system.

The joint performance of all components in the system can be indicated by vector $\mathrm{X} = (\mathrm{x}_1, \mathrm{x}_2, \dots, \mathrm{x}_\mathrm{n})$ called a state vector. Similarly, the binary variable ϕ indicates the state of the system:

Similarly, the binary variable ϕ indicates the state of the system:

$$\phi_i = \begin{cases} 1, & \text{if system i is functioning,} \\ 0, & \text{if system i is failed.} \end{cases}$$

The term binary variable will refer to a variable taking on the values 0 or 1.

The state of system is determined completely by the states of the components, so that may write:

$$\phi = \phi(x), \quad \text{where} \qquad x = (x_1, \ldots, x_n).$$

The function $\phi(x)$, is called the structure-function of the system. A knowledge of the structure-function is equivalent to an understanding of the structure of the system.

B. THE DEFINITION *K-OUT-OF-N* STRUCTURE

A system function, if and only if at least *k* of the n components is working, is called a *k-out-of-n* structure-function.

The structure function *k-out-of-n* structure can be written:

$$\phi_i = \begin{cases} 1, \text{ if } \sum_{i=1}^{n} x_i \geq k\,, \\ 0, \text{ if } \sum_{i=1}^{n} x_i \leq k. \end{cases}$$

or equivalently,

$$\phi(x) = \prod_{i=1}^{n} x_i \;\; for \quad k = n,$$

While,

$$\phi = (x_1 \ldots x_k)(x_1 \ldots x_{k-1} x_{k+1}) \ldots (x_{n-k+1} \ldots x_n)$$

$$\equiv max\{(x_1 \ldots x_k)(x_1 \ldots x_{k-1} x_{k+1}) \ldots (x_{n-k+1} \ldots x_n)\}$$

for $1 \leq k \leq n$, every choice of *k* out of the *n x's* appears exactly. A parallel structure is a *1-out-of-n* structure and a series structure is an *n-out-of-n* structure.

C. COHERENT STRUCTURES

The system with a monotone structure function is called semi-coherent. A semi-coherent system having relevant components is then called a coherent system. A system of components is coherent if all its components are relevant and the structure function is non-decreasing.

There are only two semi-coherent structures that are not coherent:

- $\phi(x) = 0$, which fails for every state of its components,
- $\phi(x) = 1$, which performs for every state of its components.

A physical system would be poorly designed if improving the performance of a component (that is, replacing a failed component with a functioning component) caused the system to deteriorate.

Assume that the system will not run worse than before if we replace a component in a failed state with one functioning. This is the same as requiring that the structure-function be non-decreasing in each argument.

D. STRUCTURES REPRESENTED BY PATHS AND CUTS

A structure of order n consists of n components numbered from *1 to n*. The set of components is denoted by:

$$c = \{1,2, \ldots n\}$$

A path set P is a set of components in C that, by functioning, ensures that the system is working. A path set is said to be minimal if it cannot be reduced without losing its status as a path set.

A critical path vector for components i is a state vector *(1$_i$; x)* such that:

$$\phi(1_i, x) = 1 \quad while \qquad \phi(0_i, x) = 0$$

This is equivalent to requiring that:

$$\phi(1_i, x) - \phi(0_i, x) = 1$$

In other words, given the states of the other components *(1$_i$; x)*, the system is functioning if and only if component *i* is working. It is, therefore, natural to call *(1$_i$; x)* a critical path vector for component *i*.

A critical path set *C(1$_i$; x)* corresponding to the critical path vector *(1$_i$; x)* for component *i* is defined by:

$$C(1_i, x) = \{i\} \cup \{j; x_j = 1, j \neq i\}$$

The total number of critical path sets (path vectors) for component *i* is:

$$\eta_\phi(i) = \sum_{(\cdot_i, x)} [\phi(1_i, x) - \phi(0_i, x)]$$

Since the $x_j's$ are binary variables and thus can take only two possible values*, 0 and 1*, the total number of state vectors $(\cdot, x) = (x_1, \cdots, x_{i-1}, \cdot, x_{i+1}, \cdots, x_n) \ is \ 2^{n-1}$.

A cut set K is a set of components in C which, by failing, causes the system to fail. A cut set is said to be minimal if it cannot be reduced without losing its status as a cut set. The association of a binary function with the arguments x_i and $i \in P_j$ with the j^{th} minimal path set P_j of a coherent structure ϕ is represent by:

$$\boldsymbol{\rho_j(x) = \prod_{i \in P_j} x_i}, \quad \textbf{(1)}$$

If all components in the j^{th} minimal path set function take the value 1, and 0 otherwise.

The structure ρ_j is the j^{th} minimal path series structure, *(J=1,...,p)* where *p* is the number of minimal path sets of ϕ and ρ_j is the structure function of a series arrangement of the components of the j^{th} path set.

The structure are working if and only if at least one of the minimal path structures is work, follow identity is true representing the structure as a parallel arrangement of the minimal path series structures:

$$\boldsymbol{\phi(x) \equiv \coprod_{j=1}^{P} \rho_j(x) \equiv \prod_{j=1}^{P} [1 - \rho_j(x)]}, \quad \textbf{(2)}$$

The structure may be interpreted as a parallel structure of the minimal path series structures. Combining equation 2 with equation 1, result is:

$$\boldsymbol{\phi(x) = \coprod_{j=1}^{P} \prod_{i \in P_j} x_i} \quad \textbf{(3)}$$

The association of a binary function with arguments $x_i, i \in K_j$ with the j^{th} minimal cut set K_j of a coherent structure ϕ is represented by:

$$\boldsymbol{\kappa_j(x) = \coprod_{i \in K_j} x_i,} \quad \textbf{(4)}$$

if all the components in the j^{th} minimal cut set fail, which takes the value 0 and 1 otherwise. The structure κ_j is the j^{th} *minimal parallel cut structure (J=1,..., K)* where *K* is the number of minimal cut sets of ϕ and $\kappa_j$is the structure function of a parallel arrangement of the components of the j^{th} cut set.

The structure fails if and only if at least one of the minimal cut structures fails; the following identity is true:

$$\boldsymbol{\phi(x) = \prod_{j=1}^{K} \kappa_j(x)} \quad \textbf{(5)}$$

representing the structure as a series arrangement of the minimal cut parallel structures.

The structure may be interpreted as a series structure of the minimal cut parallel structure. By combining eq. 4 and 5, yield:

$$\boldsymbol{\phi(x) = \prod_{j=1}^{K} \coprod_{i \in K_j} x_i} \quad \textbf{(6)}$$

E. RELIABILITY FUNCTION IN COHERENT STRUCTURES

In this section, the discussion will extend to include a probabilistic aspect of coherent structure. It is concerned about the reliability of a system based on the reliability components

Let the component state X_i be a Bernoulli random variable($P(X_i = 1) = p_i$ and $P(X_i = 0) = q_i$, where $q_i = 1 - p_i$. Then $P(X_i = 1) = p_i$ is called the reliability of component c_i and $i = 1; 2;...; n$. The corresponding system reliability is given by

$$\boldsymbol{R_\phi(P) = P\{\phi(X) = 1|p\} = E[\phi(X)|p], \qquad p = p_1, p_2, \ldots, p_n} \quad \textbf{(7)}$$

R_ϕ is called the reliability based on the structure function ϕ**.** Using the assumption of component independence, the reliability R_ϕ is wholly determined by component reliabilities $(p_1; p_2;...; p_n)$.

R_ϕ can be written as R_ϕ (P), and in the case of $(p_1 = p_2 = ... = p_n = p)$, the reliability function can be written as R_ϕ (p) from a common component reliability *p*.

When component c_i is functioning, the reliability function of a system is:

$$\boldsymbol{R_\phi(1_i, p) = P\{\phi(1_i, X) = 1|p\};} \quad \textbf{(8)}$$

and the reliability function of a system given component c_i fails is:

$$\boldsymbol{R_\phi(0_i, p) = P\{\phi(0_i, X) = 1|p\};} \quad \textbf{(9)}$$

where *p = (p1;...; pi-1, pi+1,...; pn).*

Exact system reliability can be computed using the structure function $\phi(X)$.

$$\boldsymbol{= E(\coprod_{i=1}^{t} \coprod_{\{j:z_{ij}=0\}} X_j) = E(\prod_{i=1}^{r} \coprod_{\{j:w_{ij}=0\}} X_j),} \quad \textbf{(10)}$$

where z_{ij} is the jth element of minimal-path vector z_i, and where w_{ij} is the j^{th} element of minimal-cut vector w_i and assume independent components.

F. RELIABILITY FUNCTION WITH LIFE DISTRIBUTION

A comprehensive discussion of reliability theory can be found in (Barlow and Proschan 1975) and (Kaufmann, Grouchko, and Cruon 1977). Let us consider the expression of reliability function when a system and its components have life distribution.

Let $F_i(t)$ be a life distribution of component c_i and let:

$$X_i(t) = \begin{cases} 1, & \text{if } c_i \text{ is functioning until time t,} \\ 0, & \text{if } c_i \text{ is failed before time t.} \end{cases}$$

Then, the reliability of component c_i at time t is:

$$\boldsymbol{P[X_i(t) = 1] = E[X_i(t)] = S_i(t) = 1 - F_i(t),} \quad \textbf{(11)}$$

and the system reliability at time t is:

$$\boldsymbol{R_\phi[S(t) = 1] = P\{\phi[X(t)] = 1\} = E_\phi[X_i(t)].} \quad \textbf{(12)}$$

3. THE RIGHT DATA CENSORED

The data is considered complete when the exact time of each system failure is known. In many cases, the data contain uncertainties, i.e., when the failure occurred is unknown. The data containing such uncertainty as when the event occurred are regarded as incomplete or partial. Incomplete data can be classified as censored or truncated (Gijbels, 2010).

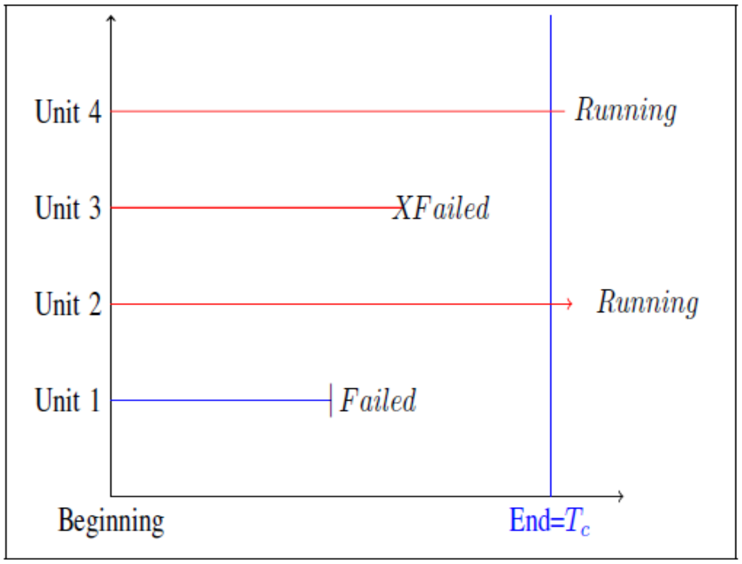
From the theoretical point of view, censoring may not be the most efficient way to conduct an experience, but due to time, cost, or practical things, it's so frequent that researchers have had to find ways to deal with it.

Characterizing the censoring mechanisms is essential to analyze the data and the phenomena in the study. Such a report can be based on several elements: the status of the entity observed, the study's span, the system's dynamic in the study, and the times of start and finish of the observations. Censoring mechanisms can also be characterized based on when and how the time to finish the study is defined. One of the most common types of censored data that may arise in real cases is type-I right censored data.

In type-I right censored data, all units of a system are observed up to the date of completion of the study. For this censorship scheme, the time each unit is under observation is fixed, while the number of units that fail (uncensored observations) is random. In this type of censoring, the stopping time (t_c) is defined or pre-established, and the number of failures observed during the analysis period is random. It ends the experiment and stops monitoring all the entities at some pre-specified time tc, independent of the event of interest. The Weibull distribution is the most popular statistical distribution in reliability engineering (Murthy et al., 2004). It can be used to fit many life distributions, and it has a significant advantage in the reliability field by changing the parameters to adjust perfectly to the reliability data.

Type I censoring occurs when the experiments are run only for a fixed duration t_c; the lifetimes are known for those whose lifetimes are $t_i \leq t_c$, as it's possible to see in Fig. 1.

FIGURE 1: Fixed type I right censored.



The difference between type I and type II is that in type I censoring, the number of observed lifetimes is a random variable, and in type II, the number of observed events is fixed.

4. SIMULATION FOR COHERENT SYSTEMS: K-OUT-OF-ON

To make a simulation Monte Carlo of complex systems k-out- of-n, using Weibull distribution, the following algorithm have been developed:

Step 1 — Define the function of the structure of the *k-out-of-n* system;

Step 2 — Calculate the time to censoring - t_c with the parameters of Weibull distribution;

Step 3 — Generate t_i from random distribution function;

Step 4 — Compare the time t_i with T_c to each component and give $x_i = 0$ if are above or $x_i = 1$ below the t_c;

Step 5 — With x_i and structure function, calculate if the system is working or not;

Step 6 — Repeat for *M* times (the dimension of the cycle simulation);

Step 7 — Calculate the reliability: the number of times the system works for the number of samples M.

The program was written in R software, and at the beginning, it defined the function structure and the rest of the parameters (number of simulations, etc.). After that, the loop “for” is applied to make the cycle where reliability is calculated in the Monte Carlo simulation core. The difference between *2-out-of-3* and the bridge example *2-out-of-5* is the structure-function of the system. This is defined in the program by a function with the name “*str fun”*.

LISTINGS 1: Simulation for *k-out-of-n* function *str fun*.

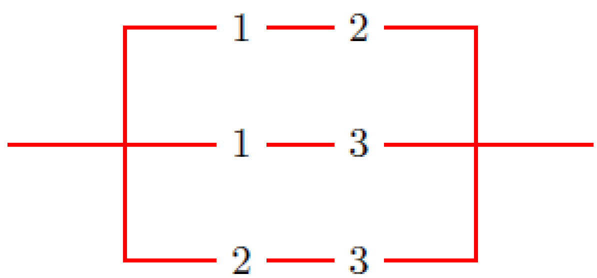
```
str_fun<-function(x1,x2,x3,x4,x5){
            res=x1*x4+x2*x5+x1*x3*x5+x2*x3*x4
           -x1*x3*x4*x5-x1*x2*x3*x5-x1*x2*x3
           *x4-x2*x3*x4*x5-x1*x2*x4*x5+2*x1
           *x2*x3*x4*x5
return(res)
   }
simul_fun<-function(m){
              cr=0
...
```

5. SIMULATION OF COHERENT SYSTEMS

A. SIMULATION OF COHERENT SYSTEM: *2-OUT-OF-3*

To illustrate the algorithm, the particular case of the *2-out-of-3* structure is considered, with structure-function given by

FIGURE 2: Coherent system *2-out-of-3*.



Note that replicating each component is only for analyses; physically, each component appears only once. Some equipment is operational if and only if at least two of its three components are working like the engines of an airplane.

Consider the *2-out-of-3*-structure in figure (2). The minimal path sets are:

$$P_1 = \{1,2\}, P_2 = \{1,3\}, P_3 = \{2,3\}$$

The minimal cut sets are:

$$K_1 = \{1,2\}, K_2 = \{1,3\}, K_3 = \{2,3\}$$

The *2-out-of-3* structure may therefore be represented as a series structure of its minimal cut parallel structures as illustrated in Figure 2.

In this particular example the number of minimal cut sets coincide with the number of minimal path sets. This will usually not be the case.

$$\boldsymbol{\phi = x_1x_2x_1x_3x_2x_3} \quad (13)$$

$$\boldsymbol{\equiv x_1x_2x_3 + x_1x_2(1 - x_3) + x_1(1 - x_2)x_3 + (1 - x_1)x_2x_3.}$$

It uses the same Weibull parameters for all components: the shape parameter β has 0,5, 1, 1,5 and 2; the scale parameter is η=10 for all components and simulations. The simulation is made for different numbers of samples to verify the impact of the number of samples for each simulation. Another important characteristic is to simulate reliability with other censored data; in this case, 5%,10%, 20% and 30% are used.

TABLE 1: Simulation *2-out-of-3*, Weibull (β;%C; n), η = 10.

	$C_{5\%}$				$C_{10\%}$				$C_{20\%}$				$C_{30\%}$			
Sample	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2
10	1	0.7	0.8	0.6	1	1	0.7	0.8	1	1	1	0.7	1	1	0.9	0.7
100	0.94	0.94	0.82	0.69	0.99	0.99	0.93	0.76	1	1	0.96	0.81	1	1	0.99	0.88
500	0.91	0.88	0.82	0.73	1	0.98	0.93	0.76	1	0.99	0.95	0.86	1	1	0.98	0.9
1000	0.93	0.88	0.81	0.74	0.99	0.97	0.89	0.78	1	1	0.96	0.82	1	1	0.98	0.86
2000	0.94	0.89	0.8	0.75	0.99	0.98	0.9	0.78	1	1	0.95	0.82	1	1	0.99	0.86

The results are explicit in Table 1. From the table analysis, it's possible to see that with the increase in the sample number, the reliability value stabilized at a specific value. With the increase of the shape factor, the value of reliability decreases; however, the most unexpected evolution data, β, is the growth of reliability with the increase of censored data, when it was expected to decrease. One possible explanation is that with the rise of censorship, the system ends up better, vanishing the faults and giving globally less damage.

Testing with other parameters and comparative tests with different parameters for each component will be recommended.

B. SIMULATION FOR *2-OUT-OF-5* BRIDGE STRUCTURE

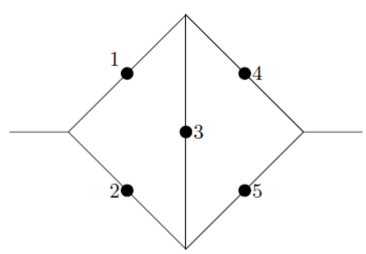
Consider a bridge structure such as that given by the physical network in Figure 3. The minimal path sets are:

$$P_1 = \{1,4\}, P_2 = \{2,5\}, P_3 = \{1,3,5\}, P_4 = \{2,3,4\}$$

The minimal cut sets are:

$$K_1 = \{1,2\}, K_2 = \{4,5\}, K_3 = \{1,3,5\}, K_4 = \{2,3,4\}$$

FIGURE 3: Bridge Structure Diagram.



Using the equation 2 and the minimal path sets, the bridge structure may be represent as a parallel-series diagram.

$$\begin{aligned} \rho_1 &= x_1 \cdot x_3 \cdot x_5 \quad (14) \\ \rho_2 &= x_2 \cdot x_3 \cdot x_4 \\ \rho_3 &= x_1 \cdot x_4 \\ \rho_4 &= x_2 \cdot x_5 \end{aligned}$$

accordingly, the structure function may be written:

$$
\begin{aligned}
\phi(x) &= \coprod_{j=1}^{4} \rho_j(x) = 1 - \prod_{j=1}^{4}\big(1-\rho(x)\big) \\
&= 1-\big(1-\rho_1(x)\big)\big(1-\rho_2(x)\big)\big(1-\rho_3(x)\big)\big(1-\rho_4(x)\big) \\
&= x_1x_4 + x_2x_5 + x_1x_3x_5 + x_2x_3x_4 - x_1x_3x_4x_5 \\
&= x_1x_4 + x_2x_5 + x_1x_3x_5 + x_2x_3x_4 - x_1x_3x_4x_5 \\
&\quad -x_1x_2x_3x_5 - x_1x_2x_3x_4 - x_2x_3x_4x_5 - x_1x_2x_4x_5 \\
&\quad +2x_1x_2x_3x_4x_5
\end{aligned}
$$

Similarly, using equation 5 and the minimal cut sets, we may represent the bridge as a series-parallel structure:

$$
\begin{aligned}
\kappa_1 &= 1-(1-x_1)(1-x_2) \\
\kappa_2 &= 1-(1-x_4)(1-x_5) \\
\kappa_3 &= 1-(1-x_1)(1-x_3)(1-x_5) \\
\kappa_4 &= 1-(1-x_2)(1-x_3)(1-x_4)
\end{aligned}
$$

It used the same Weibull parameters to all components and is similar *with 2-out-of-3:* the shape parameter β has the values 0,5; 1, 1,5 and 2, and the scale parameter is $\eta=10$. The simulation is made for different numbers of samples on various percentages of censored data to verify the impact of the number of samples for each simulation.

TABLE 2: Simulation *2-out-of-5*, Weibull (β;%C; n), η = 10.

	$C_{5\%}$				$C_{10\%}$				$C_{20\%}$				$C_{30\%}$			
Sample	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2	$\beta_{0.5}$	β_1	$\beta_{1.5}$	β_2
10	0.9	0.9	0.7	0.7	1	1	0.8	0.6	1	1	1	0.8	1	1	1	0.9
100	0.92	0.89	0.79	0.71	1	0.95	0.88	0.81	1	0.99	0.97	0.88	1	1	1	0.83
500	0.94	0.9	0.82	0.74	0.99	0.98	0.91	0.82	1	1	0.96	0.85	1	1	0.98	0.89
1000	0.93	0.9	0.83	0.77	0.99	0.97	0.91	0.8	1	1	0.95	0.86	1	1	0.99	0.91
2000	0.93	0.9	0.83	0.76	1	0.98	0.91	0.8	1	1	0.97	0.84	1	1	0.99	0.88

The results are explicit in Table 2 compared with the simulation of reliability *2-out-of-3* system; the evolution and the pattern are the same. The absolute values of reliability for each simulation are very close. However, this structure is more complex and has almost the same redundancy as the simple *2-out-of-3* system.

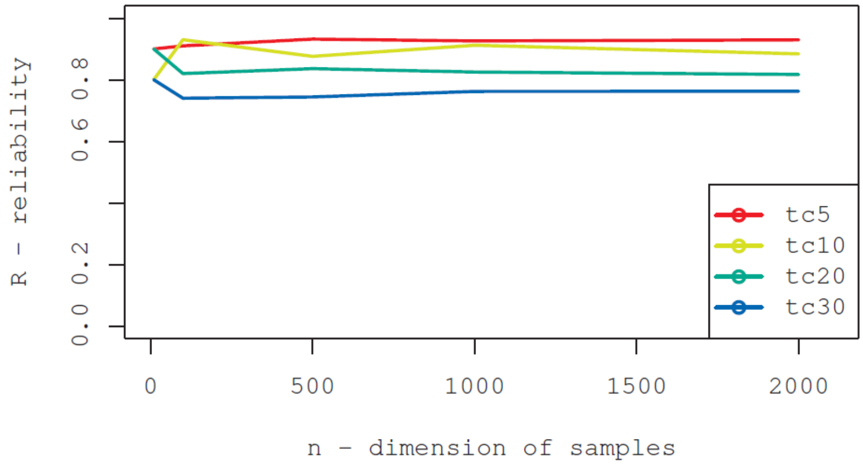
6. CONCLUSIONS AND OUTLOOK

The subject of reliability and simulation of systems and the structural relationship between a system and its components is very important in the field of reliability. Reliability Block Diagrams — RBD can be defined as a network of blocks describing the system's function with logical connections of components needed to produce a specified system function.

Survival testing and reliability studies are usually focused on estimating an unknown cumulative distribution function (CDF). In simulation studies, it's normal to use computational power to test particular hypotheses and assess the validity and accuracy of various statistical methods or procedures concerning a known truth. These procedures and algorithms provide an empirical estimation of the sampling distribution of the parameters of interest.

The complex and coherent systems' theory and the reliability functions are derived, and algorithms and simulations with censored data are carried out with a new and original approach.

FIGURE 4: Reliability, Weibull (β;C=10% and n=50).



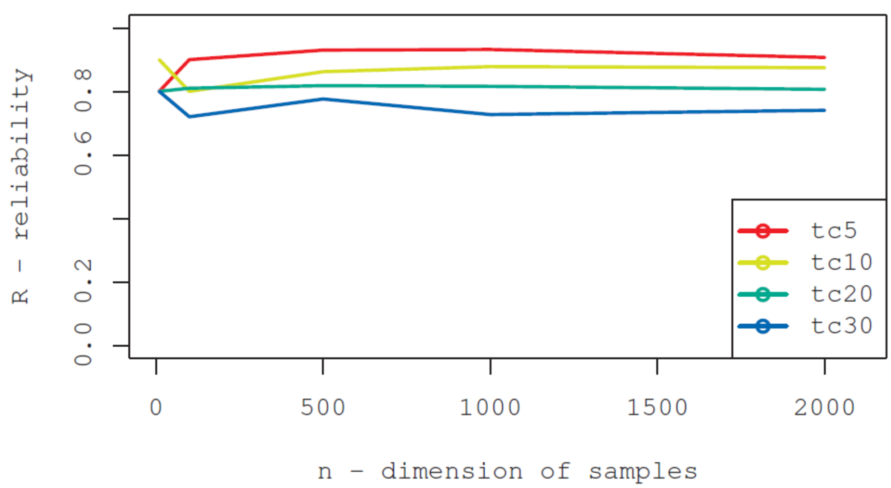
In Figures 4 and 5, which summarize in a condensed and graphic form the results of Tables 1 and 2, it can be seen that there is not an equal pattern for all shape factors but rather a tendency to approach the value as the percentage of censored data increases. That is, the more censored data there is in the sample, the more bias and more error, relative to the true value, the sample has. The behaviour of factors β *= 0,5* and β *= 1,5* are similar but with different scales.

This paper intends to help develop the best procedures to simulate the reliability field in a complex and coherent system by generating a sample of data with a particular characteristic (right censored and type I).

In conclusion, the work intends to generate more discussion and attention to the algorithms that simulate data censored with complex and coherent systems and give some tools and results to make the simulations and the studies more accurate and optimized.

The next step for this work would be to continue the study with the same algorithm for the other statistical distributions, namely the exponential, gamma, log-normal, and normal distributions. Another important step would be to verify whether the chosen algorithm was well adapted to other types of censored data, as would be the case with type II censored data. Finally, as this work was developed in a specific software language, the R software, it would also be interesting to verify the performance of algorithms in other languages, such as Python or C++.

FIGURE 5: Reliability, Weibull (β;C=10% and n=50).



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DECLARAÇÃO ÉTICA

CONFLITO DE INTERESSE: Nada a declarar. **FINANCIAMENTO:** Nada a declarar. **REVISÃO POR PARES:** Dupla revisão anónima por pares.